Actuator Failure Detection in the Control of Distributed Systems

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A method is presented for the detection of actuator failures in the control of distributed-parameter systems. The method can be used with any control scheme and is based on identifying the actuator input from the system response by modal analysis. The failure detection can be carried out as an on- or off-line operation. First, the modal excitations are identified. Estimates of the external forces are then synthesized as a linear combination of the modal forces. The identified external forces are compared with the actuator commands to detect the failure and isolate the faulty component(s). The effects of actuator failure on the performance of control systems are investigated, as well as factors that affect the reliability of the failure detection, such as measurement noise and observation spillover. A guideline is proposed to locate the actuators in such a way as to aid the failure detection process.

Introduction

A VERY important problem in the design of a control system is the reliability of the components carrying out the control action and the effect of the failure of one or more components on the performance of the control system. Intuitively, one expects the overall performance to degrade. The amount of degradation is closely related to the number of components and to the control method. In some cases, even instabilities may occur. Because of this, any failure of an actuator or sensor must be detected immediately after failure and corrective action must be taken within a very short amount of time.

This paper is concerned with the effects of actuator failure and the detection of such failures in the control of distributed-parameter systems. Most of the existing failure detection schemes deal with low-order models where the number of sensors and actuators used to implement the control law is very small.[1-5] A survey of such methods is given in Ref. 5. In distributed systems, such as large spacecraft, a large number of components is generally required to obtain an effective control action. A majority of the methods proposed for failure detection in low-order dynamical systems thus become inapplicable with distributed systems.

The problem of failure detection and the isolation of faulty components in the control of large structures has recently begun to receive interest.[6-8] Reference 6 summarizes the methods commonly used in failure detection and discusses their applicability to distributed structures. The methods used in Ref. 6 to detect failure are based on parity relations. By using some of the redundancies in the control system, one can write parity relations that should be equal to zero under normal operation. In the event of failure, the parity relations no longer equal zero, indicating a faulty component. The location of an actuator is also important in analyzing the effect of failure and isolating the faulty actuator.[7,8] Note that not only is the reliability of a control system important in the event of failure, but also that the accuracy of the fault detection is critical. Factors such as measurement and actuator noise, parameter uncertainties, and observation spillover directly affect the reliability of the failure detection scheme. The effects of such factors have been analyzed for low-order systems.[9,10] For example, a small amount of deviation from the desired behavior of an actuator may be acceptable based on the prescribed reliability level, but this deviation may be amplified by the failure detection algorithm.

In this paper, the effects of actuator failure are analyzed for different feedback control techniques. Here, colocated control[11] is found to be superior from a reliability point of view. Two methods of extracting the modal coordinates from the system output are considered, namely, modal filters[12] and observers.[13] It is shown that, when observers are used, not only do the closed-loop poles of the control system change in the event of failure, but the convergence of the observer is not guaranteed and the separation principle is no longer valid. This second drawback is not pertinent to modal filters, because their implementation does not require knowledge of the external excitation. The conclusion is that modal filters are more desirable to use in distributed-parameter control systems.

A method is also developed in this paper to detect actuator failures for distributed systems. The method is applicable to any control scheme and is based on identifying the actuators input by analyzing the response of the modal coordinates. Plots of the identified actuator forces (or torques) are compared with the commands given to the actuators. A substantial difference indicates failure. Considered are the effects of sensor and actuator noise and the consequences of not being able to extract the modal coordinates from the system output accurately. The failure detection can be carried out as an on- or off-line procedure. It is assumed that an accurate mathematical model is available.

Note that identifying the actuator forces from the system output and comparing them with the actuator commands is similar to developing parity relations for each actuator. Also, more than one such relation is developed for each actuator, so that a kind of "voting" is used in detecting failure. Guidelines are proposed to locate the actuators in a way to aid the failure detection process. The methods described in this paper are envisioned as being parts of a sequential decision tree to identify and isolate failure and to assess the reliability of control systems.

System Equations

A large number of dynamical systems can be modeled mathematically as distributed-parameter systems. Among these are large flexible structures, chemical processes, and thermal systems. We consider here the class of distributed systems

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described by the general equation

$$m(x)\ddot{u}(x,t) + Lu(x,t) = f(x,t) \qquad (1)$$

where $u(x,t)$ is the displacement at spatial coordinate x at time $t, m(x)$ a weighting function (representing the mass distribution for structures), L a linear differential self-adjoint operator of order $2p$ (denoting the stiffness), and $f(x,t)$ the external excitation including controls. It is assumed that $u(x,t)$ and $f(x,t)$ are in a Hilbert space H with an inner product $\langle , \rangle$ and a corresponding norm $\| \ \|$. The displacement $u(x,t)$ is subject to the boundary conditions $B_i u(x,t) = 0$, where B_i are boundary operators and $i = 1,2,\ldots,p$.

It follows that the system of Eq. (1) admits an infinite and countable set of eigenvalues λ_r related to the natural frequencies by $\lambda_r = \omega_r^2$ $(r = 1,2,\ldots)$ and corresponding eigenfunctions $\phi_r(x)$ $(r = 1,2,\ldots)$. The eigenfunctions can be normalized to yield the orthogonality relations $\langle \phi_r(x), m(x)\phi_s(x)\rangle = \delta_{rs}$, $\langle \phi_r(x), L\phi_s(x)\rangle = \lambda_r \delta_{rs}$, where δ_{rs} is the Kronecker delta and $r,s = 1,2,\ldots$.

We can expand $u(x,t)$ and $f(x,t)$ as

$$u(x,t) = \sum_{r=1}^{\infty} \phi_r(x) u_r(t) \qquad (2a)$$

$$f(x,t) = \sum_{r=1}^{\infty} m(x)\phi_r(x) f_r(t) \qquad (2b)$$

where $u_r(t)$ and $f_r(t)$ are modal coordinates and modal forces, respectively. Using Eqs. (2) and the orthogonality relations, we obtain

$$u_r(t) = \langle u(x,t), m(x)\phi_r(x)\rangle \qquad (3a)$$

$$f_r(t) = \langle f(x,t), \phi_r(x)\rangle \qquad (3b)$$

Equations (2) and (3) constitute the so-called expansion theorem.[14] Introducing Eqs. (2) into Eq. (1) and using Eqs. (3), we arrive at the modal equations of motion in the form

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t), \quad r = 1,2,\ldots \qquad (4)$$

For control systems, most methods use the displacement and velocity of the distributed system as feedback, so that, if the external excitation is in the form of control inputs, we can write

$$f(x,t) = f[u(x,t), \dot{u}(x,t)] \qquad (5)$$

Distributed measurements and controls are not within the state-of-the-art, so we consider here implementation by discrete components. Equation (5) is then replaced by

$$\boldsymbol{F}(t) = \boldsymbol{F}[\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t)] \qquad (6)$$

where

$$\boldsymbol{F}(t) = [F_1(t) \quad F_2(t) \quad \cdots \quad F_m(t)]^T \qquad (7a)$$

$$\boldsymbol{y}(t) = [u(x_1,t) \quad u(x_2,t) \quad \cdots \quad u(x_k,t)]^T \qquad (7b)$$

denote the external forces (or moments) and sensors measurements, respectively, and where m is the number of inputs and k the number of sensors.

When determining $\boldsymbol{F}(t)$, two main approaches emerge for control systems. The first is to relate the control forces to the system output directly.[11] Generally, this form of control requires that the actuators and sensors be colocated and has come to be known as colocated control or direct velocity feedback. The control law has the form

$$F_j(t) = G_j \dot{y}_j(t) + H_j y_j(t), \qquad j = 1,2,\ldots,m \qquad (8)$$

where G_j and H_j are the feedback gains.

The second form of control is by synthesizing the modal coordinates $u_r(t)$ and $\dot{u}_r(t)$ from $y(t)$ and $\dot{y}(t)$ and using a modal control law. Extraction of the modal coordinates from the system output can be achieved by Luenberger observers,[13] temporal filters, or modal filters.[12]

The Effects of Actuator Failure

In most systems, failure of an actuator during control degrades system performance. Sometimes, instabilities may occur. To investigate the effects of actuator failure, we first express the actual system input, denoted by $\boldsymbol{F}_a(t)$ as

$$\boldsymbol{F}_a(t) = S\boldsymbol{F}(t) + \boldsymbol{v}(t) \qquad (9)$$

where S is a diagonal matrix having the form

$$S = \mathrm{diag}[S_1 \quad S_2 \quad \cdots \quad S_m] \qquad (10)$$

where S_j $(j = 1,2,\ldots,m)$ denotes the level of failure in the jth actuator. For example, a faulty actuator may impart only half the desired input, in which case, $S_j = 0.5$. Sometimes a certain amount of failure like this may be acceptable from a reliability point of view. Note that S_j $(j = 1,2,\ldots,m)$ can be time varying as well. $\boldsymbol{v}(t)$ is a vector, such that

$$\boldsymbol{v}(t) = [v_1(t) \quad v_2(t) \quad \cdots \quad v_m(t)]^T \qquad (11)$$

where $v_j(t)$ $(j = 1,2,\ldots,m)$ describes the erratic behavior, if any, of the jth actuator, including actuator noise. It is assumed here that this noise can be described by a Gaussian stochastic process.

In some control schemes, one needs to extract part of the modal coordinates from the system output, because the control method is based on controlling the modal coordinates. Here, we wish to examine the effects of actuator failure on this process of extraction of modal coordinates. We first consider Luenberger observers, which are described by the equation

$$\dot{\hat{\boldsymbol{w}}}(t) = A\hat{\boldsymbol{w}}(t) + B'\boldsymbol{F}(t) + K[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)] \qquad (12)$$

where $\hat{\boldsymbol{w}}(t)$ is the estimate of the state vector $\boldsymbol{w}(t)$ and the motion in the state space is described by

$$\dot{\boldsymbol{w}}(t) = A\boldsymbol{w}(t) + B'\boldsymbol{F}(t) \qquad (13)$$

where A is the state matrix containing the natural frequencies and B' the input influence matrix whose elements are dependent on the locations of the external disturbances. $\hat{\boldsymbol{y}}(t)$ is the estimate of the system output $\boldsymbol{y}(t)$, which is related to the system state by

$$\boldsymbol{y}(t) = D\boldsymbol{w}(t) \qquad (14)$$

The elements of D depend on the kind and location of the sensors. K is the observer gain matrix determined by the analyst.

It is clear that in distributed-parameter systems the order of the system has to be truncated to a finite one for implementation of the Luenberger observers. The state vector $\boldsymbol{w}(t)$ is generally chosen as $\boldsymbol{w}(t) = [u_1 \quad \dot{u}_1 \quad u_2 \quad \dot{u}_2 \quad \cdots \quad u_n \quad \dot{u}_n]^T$, where n is the number of controlled modes. The consequences of using an observer of finite order to control a distributed system is considered in Ref. 15 and is shown to lead to observation spillover from uncontrolled modes that may have destabilizing effects under certain circumstances.

Let us define by $\boldsymbol{e}(t)$ the error in the observation process, so that $\boldsymbol{e}(t) = \boldsymbol{w}(t) - \hat{\boldsymbol{w}}(t)$. Assuming a linear control law in the form $F(t) = G\hat{w}(t)$, where G is the control gain matrix, the closed-loop equations can be written as

$$\begin{bmatrix} \dot{w}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A+B'G & -B'G \\ 0 & A-KD \end{bmatrix} \begin{bmatrix} w(t) \\ e(t) \end{bmatrix} \tag{15}$$

In the event of actuator failure, the control vector $F(t)$ in Eq. (13) is replaced by the actual input vector $F_a(t)$. Equations (15) then become

$$\begin{bmatrix} \dot{w}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A+B'SG & -B'SG \\ -B'(I-S)G & A-KD+B'(I-S)G \end{bmatrix} \times \begin{bmatrix} w(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B'v(t) \\ B'v(t) \end{bmatrix} \tag{16}$$

Clearly, failure of an actuator affects both the observer and system closed-loop poles. In addition, the deterministic separation principle is no longer valid, so that a rational assessment of the closed-loop system and observer poles becomes difficult. For low levels of failure, one can analyze the change in the poles of the system by a perturbation approach.

When modal filters are used, the sensors output is interpolated to obtain estimates $\bar{u}(x,t)$ of the distributed profile $u(x,t)$. Then, this estimate is used in conjunction with Eq. (3a) to yield estimates of the modal coordinates.[12] Because of computational considerations, it has become customary to select an interpolation process where the time and space dependency of the interpolation are separate,[12] so that the estimate of the distributed profile can be expressed as

$$\bar{u}(x,t) = \sum_{j=1}^{k} C(x,x_j)\left[u(x_j,t) + q_j(t)\right] \tag{17}$$

where $C(x,x_j)$ are interpolation functions, x_j the sensors locations, and $q_j(t)$ $(j=1,2,\ldots,k)$ the sensor noise (also assumed to be Gaussian). A similar equation can be written for velocity measurements. Introducing Eq. (17) into Eq. (3a), we obtain estimates of the modal coordinates in the form

$$\bar{u}_r(t) = \langle \bar{u}(x,t), m(x)\phi_r(x)\rangle = \sum_{j=1}^{k} C_{rj}\left[y_j(t) + q_j(t)\right] \tag{18}$$

where

$$C_{rj} = \langle C(x,x_j), m(x)\phi_r(x)\rangle, \quad r=1,2,\ldots;\ j=1,2,\ldots,k \tag{19}$$

are modal filter gains and can be computed off-line. It can be shown[12] that the error in the estimates of the modal coordinates has the form

$$e(t) = u(t) - \bar{u}(t) = [1-CD]u(t) - p(t) \tag{20}$$

where

$$e(t) = [e_1(t) \quad e_2(t) \quad \cdots]^T \tag{21a}$$

$$\bar{u}(t) = [\bar{u}_1(t) \quad \bar{u}_2(t) \quad \cdots]^T \tag{21b}$$

$$p(t) = [p_1(t) \quad p_2(t) \quad \cdots]^T \tag{21c}$$

$$p_r(t) = \sum_{j=1}^{k} C_{rj} q_j(t) \tag{21d}$$

and $e_j(t) = u_j(t) - \bar{u}_j(t)$, $(j=1,2,\ldots)$. D is defined such that $D_{jr} = \phi_r(x_j)$ and relates the sensor locations and modal coordinates in the form $y(t) = Du(t)$. Note that the error vector $e(t)$ and matrix D used for modal filters are different from the ones used for Luenberger observers.

We observe from Eqs. (17-21) that implementation of modal filters does not require knowledge of the external forces, so that modal filters are not affected by actuator failure. Because of this, they are more desirable to use than the Luenberger observers. It should be noted that, in general, observers require a smaller number of sensors to extract the system state than modal filters. However, from Eq. (16), we conclude that whatever mechanism is used to extract the modal coordinates from the system output, the control system may be destabilized in the event of actuator failure.

Another common form of control, which is not based on modal analysis, is colocation.[11] While there are numerous variants of this technique, the basic form is the same and is described by Eq. (8), which can be written in the matrix form as

$$F(t) = G\dot{y}(t) + Hy(t) \tag{22}$$

where G and H are diagonal matrices containing the control gains G_j and H_j $(j=1,2,\ldots,m)$. The discrete force inputs can be treated as distributed by writing

$$f(x,t) = \sum_{j=1}^{m} F_j(t)\delta(x-x_j^a) \tag{23}$$

where the external forces $F_j(t)$ act at $x = x_j^a$ $(j=1,2,\ldots,m)$. Substituting Eq. (23) into Eq. (3b), we obtain the modal excitation for each mode in the form

$$f_r(t) = \sum_{j=1}^{m} F_j(t)\langle\delta(x-x_j^a),\phi_r(x)\rangle = \sum_{j=1}^{m}\phi_r(x_j^a)F_j(t) \tag{24}$$

Denoting by $f(t)$ the modal force vector, such that

$$f(t) = [f_1(t) \quad f_2(t) \quad \cdots]^T \tag{25}$$

we can write the relation between the modal force vector and actual force vector in the form

$$f(t) = BF(t) \tag{26}$$

where $B_{rj} = \phi_r(x_j^a)$. It follows that for colocated control $B = D^T$, so that the closed-loop equations can be written as

$$\ddot{u}(t) + \Lambda u(t) = BGB^T\dot{u}(t) + BHB^Tu(t) \tag{27}$$

where Λ is a diagonal matrix having the form $\Lambda = \mathrm{diag}[\omega_r^2]$ $(r=1,2,\ldots)$. Because the matrix products on the right side of Eq. (27) are negative semidefinite, colocated control is guaranteed to be stable.[11] In the event of actuator failure, Eq. (27) becomes

$$\ddot{u}(t) + \Lambda u(t) = BSGB^T\dot{u}(t) + BSHB^Tu(t) \tag{28}$$

so that if the failure coefficients S_j remain positive, the closed-loop system is not destabilized. This feature of colocated control makes it more reliable than other methods in the event of actuator failure.

Identification of Faulty Components

We propose to detect failure by identifying the external excitation and comparing the identified excitation with the commands given to the actuators. The approach in identifying the external excitation is based on modal analysis. First, the modal forces $f_r(t)$ will be identified for each mode. The identified modal forces will then be synthesized to estimate the actuator inputs. From Eq. (2b), we observe that if each and every modal excitation $f_r(t)$ is identified, $f(x,t)$ can be identified at every point on the distributed system. However, given a finite number of sensors, one can extract accurately only a limited number of coordinates from the system output. This is not an important drawback, because the actuator forces we wish to identify are in the form of point inputs.

First consider the identification of the modal forces. From Eq. (4), the response of a certain mode can be expressed as

$$u_r(t) = u_{r0}\cos\omega_r t + \dot{u}_{r0}\sin\omega_r t/\omega_r + \int_0^t f_r(t-\tau)\,h_r(\tau)\,\mathrm{d}\tau \tag{29}$$

where $h_r(t) = \sin(\omega_r t)/\omega_r$ $(r=1,2,\ldots)$ is the impulse response and u_{r0} and $\dot{u}_{r0}$ the initial conditions. We assume that the external forces remain constant during each sampling period, so that Eq. (29) can be replaced with

$$u_r(kT+T) = u_r(kT)\cos\omega_r T + \dot{u}_r(kT)\sin(\omega_r T)/\omega_r + f_r(kT)(1-\cos\omega_r T)/\omega_r^2,\ k=0,1,\ldots;\ r=1,2,\ldots \tag{30}$$

where T is the sampling time. Differentiating Eq. (29) with respect to time and using external excitations constant during a sampling period yields

$$\dot{u}_r(kT+T) = -u_r(kT)\,\omega_r\sin\omega_r T + \dot{u}_r(kT)\cos\omega_r T + f_r(kT)\sin(\omega_r T)/\omega_r \tag{31a}$$

$$\ddot{u}_r(kT+T) = -u_r(kT)\,\omega_r^2\cos\omega_r T - \dot{u}_r(kT)\,\omega_r\sin\omega_r T + f_r(kT)\cos\omega_r T \tag{31b}$$

In order to identify $f_r(kT)$ from the measurements of $u_r(kT+T)$, $\dot{u}_r(kT+T)$, $\ddot{u}_r(kT+T)$, $u_r(kT)$, $\dot{u}_r(kT)$, and $\ddot{u}_r(kT)$, we rearrange Eqs. (30) and (31) in the form

$$f_r(kT) = \left[u_r(kT+T)\,\omega_r^2 - u_r(kT)\,\omega_r^2\cos\omega_r T - \dot{u}_r(kT)\,\omega_r\sin\omega_r T\right]/(1-\cos\omega_r T) \tag{32a}$$

$$f_r(kT) = \frac{\dot{u}_r(kT+T)\,\omega_r}{\sin\omega_r T} + u_r(kT)\,\omega_r^2 - \frac{\dot{u}_r(kT)\,\omega_r\cos\omega_r T}{\sin\omega_r T} \tag{32b}$$

$$f_r(kT) = \frac{\ddot{u}_r(kT+T)}{\cos\omega_r T} + u_r(kT)\,\omega_r^2 + \frac{\dot{u}_r(kT)\,\omega_r\sin\omega_r T}{\cos\omega_r T} \tag{32c}$$

Depending on the types of measurements available, one can use any one of Eqs. (32) to identify $f_r(kT)$ and, in the case of no measurement noise and no interpolation error (or no observation spillover when observers are used), all three equations yield the same result. However, only estimates of the modal coordinates are available. In order to investigate these effects, we replace u_r, $\dot{u}_r$, and $\ddot{u}_r$ in Eqs. (32) by $\bar{u}_r$, $\bar{\dot{u}}_r$, and $\bar{\ddot{u}}_r$. From Eq. (20) we obtain

$$\hat{f}_r(kT) = f_r(kT) + \frac{s_r(kT+T)\,\omega_r^2 - s_r(kT)\,\omega_r^2\cos\omega_r T - \dot{s}_r(kT)\,\omega_r\sin\omega_r T}{1-\cos\omega_r T} \tag{33a}$$

$$\hat{f}_r(kT) = f_r(kT) + \frac{\dot{s}_r(kT+T)\,\omega_r}{\sin\omega_r T} + s_r(kT)\,\omega_r^2 - \frac{\dot{s}_r(kT)\,\omega_r\cos\omega_r T}{\sin\omega_r T} \tag{33b}$$

$$\hat{f}_r(kT) = f_r(kT) + \frac{\ddot{s}_r(kT+T)}{\cos\omega_r T} + s_r(kT)\,\omega_r^2 + \frac{\dot{s}_r(kT)\,\omega_r\sin\omega_r T}{\cos\omega_r T} \tag{33c}$$

where $\hat{f}_r(kT)$ is the estimate of $f_r(kT)$ and $s_r(kT) = p_r(kT) - e_r(kT)$. Note that $\dot{s}_r(t) \neq \mathrm{d}s_r(t)/\mathrm{d}t$. It is clear that the estimates of $f_r(kT)$ vary for different kinds of sensors because the measurement noise and interpolation error are multiplied by different coefficients. In Eqs. (33a) and (33b) the coefficients are the highest, so these equations are not desirable. Equation (33c) requires modal displacements, velocities, and accelerations, which may strain the hardware resources.

The objective in identification of actuator failures here is to develop a scheme to estimate $f_r(t)$ in which the amplitudes of error associated with extraction of modal coordinates and measurement noise will not be amplified, and fewer types of measurements will be used. To this end, we propose to seek linear combinations of the system response. Introducing Eq. (32c) into Eqs. (30-31) we obtain

$$u_r(kT+T) = u_r(kT) + \frac{\dot{u}_r(kT)\sin\omega_r T}{\omega_r\cos\omega_r T} + \frac{\ddot{u}_r(kT+T)(1-\cos\omega_r T)}{\omega_r^2\cos\omega_r T} \tag{34a}$$

$$\dot{u}_r(kT+T) = \frac{\dot{u}_r(kT)}{\cos\omega_r T} + \frac{\ddot{u}_r(kT+T)\sin\omega_r T}{\omega_r\cos\omega_r T} \tag{34b}$$

Changing the time instant from kT to $kT-T$ in Eqs. (34) and substituting the result into Eq. (32c) yields

$$f_r(kT) = u_r(kT-T)\,\omega_r^2 + \dot{u}_r(kT-T)\cos\omega_r T\left(1+\frac{1}{\cos\omega_r T}\right) + \ddot{u}_r(kT)\left[\frac{(1-\cos\omega_r T)}{\cos\omega_r T} + \left(\frac{\sin\omega_r T}{\cos\omega_r T}\right)^2\right] + \frac{\ddot{u}_r(kT+T)}{\cos\omega_r T} \tag{35}$$

From Eq. (32c) we can write

$$u_r(kT-T)\,\omega_r^2 + \frac{\dot{u}_r(kT-T)\,\omega_r\sin\omega_r T}{\cos\omega_r T} = f_r(kT-T) - \frac{\ddot{u}_r(kT)}{\cos\omega_r T} \tag{36}$$

which when substituted into Eq. (35) results in

$$f_r(kT) = f_r(kT-T) + \frac{\dot{u}_r(kT)\,\omega_r\sin\omega_r T}{\cos\omega_r T} - \ddot{u}_r(kT) + \frac{\ddot{u}_r(kT+T)}{\cos\omega_r T},\ r=1,2,\ldots;\ k=1,2,\ldots \tag{37}$$

We note from Eq. (37) that only velocity and acceleration measurements are required to identify the modal forces. Furthermore, the coefficients multiplying $\dot{u}_r$ and $\ddot{u}_r$, which subsequently multiply the measurement noise and other errors, are of order $\omega_r\sin\omega_r T/\cos\omega_r T$, 1, and $1/\cos\omega_r T$. When the sampling period is small, a small angles assumption can be made, so that the order of the coefficients in Eq. (37) can be approximated as $\omega_r^2 T$, 1, and 1. It is clear that the effects of measurement noise and error associated with extraction of modal coordinates are not amplified.

We also observe from Eq. (37) that in order to begin the identification process an initial estimate of $f_r(t)$ is required. However, the error associated with not knowing this initial estimate does not propagate with time as long as the mathematical model of the distributed system is accurate. Erroneous information about the initial estimates implies that the exact amplitudes of the modal forces cannot be identified, i.e., they can be identified only to within an unknown constant. This is not an important drawback, because we are interested in comparing plots of the identified forces and the commands going into the actuators. Comparing the general shapes of these curves indicates failure. In addition, the control forces will generally be in the form of sinusoidal functions (because they are proportional to the displacements and velocities), so that their exact amplitudes can be determined with relative accuracy.

The effects of measurement noise and interpolation error (for modal filters), however, propagate with time. To investigate this propagation, we examine the difference between the identified and actual modal forces for different time instances. For $k=1$ and 2, we obtain

$$f_r(T)-\hat{f}_r(T)=f_r(0)-\hat{f}_r(0)+\frac{\dot{s}_r(T)\,\omega_r\sin\omega_r T}{\cos\omega_r T}-\ddot{s}_r(T)+\frac{\ddot{s}_r(2T)}{\cos\omega_r T} \tag{38a}$$

$$f_r(2T)-\hat{f}_r(2T)=f_r(T)-\hat{f}_r(T)+\frac{\dot{s}_r(2T)\,\omega_r\sin\omega_r T}{\cos\omega_r T}-\ddot{s}_r(2T)+\frac{\ddot{s}_r(3T)}{\cos\omega_r T},\quad r=1,2,\ldots \tag{38b}$$

Substitution of Eq. (38a) into Eq. (38b) and extension into the kth time step yields

$$f_r(kT)-\hat{f}_r(kT)=f_r(0)-\hat{f}_r(0)+\sum_{j=1}^{k}\left[\frac{\dot{s}_r(jT)\,\omega_r\sin\omega_r T}{\cos\omega_r T}-\ddot{s}_r(jT)+\frac{\ddot{s}_r(jT+T)}{\cos\omega_r T}\right] \tag{39}$$

Making a small angles assumption, Eq. (39) becomes

$$f_r(kT)-\hat{f}_r(kT)=f_r(0)-\hat{f}_r(0)-\ddot{e}_r(kT+T)+\ddot{e}_r(0)+n_r(kT)-\sum_{j=1}^{k}\frac{\dot{e}_r(jT)\,\omega_r\sin\omega_r T}{\cos\omega_r T} \tag{40}$$

where the effects of measurement noise have been combined into a single term $n_r(kT)$, which is a Gaussian stochastic process, because it is a linear combination of Gaussian stochastic processes.

Next, for the case when modal filters are used, let us examine the nature of the interpolation error that accumulates in Eq. (40). A similar analysis can be performed when Luenberger observers are used. Interpolation error is caused by a lack of a sufficient number of sensors to implement modal filters, resulting in only partial elimination of the contributions of the higher modes when extracting a certain mode from the system output. From Eq. (14) we can write

$$\dot{e}_r(t)=\sum_{j=1}^{\infty}g_{rj}\dot{u}_j(t),\quad r=1,2,\ldots \tag{41}$$

When distributed measurements are available, $g_{rj}=0$ ($r, j=1,2,\ldots$). When discrete sensors are used, g_{rj} have nonzero terms. Because $\dot{u}_j(t)$ ($r=1,2,\ldots$) are in the form of sinusoidals, we conclude that $\dot{e}_r(t)$ are also sinusoidals. It follows that summation of their amplitudes over time yields yet another sinusoidal, so that the last term in Eq. (40) does not grow without bound as time unfolds. Moreover, because the coefficients g_{rj} are known in advance, the contribution of the modes not filtered completely can be eliminated out from the estimate of $f_r(t)$ by using a low-pass filter.[16] On the other hand, using a low-pass filter introduces a phase shift into the modal coordinates.

Let us now consider the case when the same control force is applied for two sampling periods. The relations used for identifying the modal forces can then be written as

$$f_r(kT)=f_r(kT-T)+\frac{\dot{u}_r(kT)\,\omega_r\sin\omega_r T}{\cos\omega_r T}-\ddot{u}_r(kT)+\frac{\ddot{u}_r(kT+T)}{\cos\omega_r T},\quad r=1,2,\ldots;\ k=2,4,\ldots \tag{42a}$$

$$0=\frac{\dot{u}_r(kT)\,\omega_r\sin\omega_r T}{\cos\omega_r T}-\ddot{u}_r(kT)+\frac{\ddot{u}_r(kT+T)}{\cos\omega_r T},\quad k=1,3,\ldots \tag{42b}$$

Subtracting Eq. (42b) from Eq. (42a) (and adjusting the time steps), we obtain

$$f_r(kT)=f_r(kT-2T)+\frac{[\dot{u}_r(kT)-\dot{u}_r(kT-T)]\,\omega_r\sin\omega_r T}{\cos\omega_r T}-[\ddot{u}_r(kT)-\ddot{u}_r(kT-T)]+\frac{[\ddot{u}_r(kT+T)-\ddot{u}_r(kT)]}{\cos\omega_r T},\quad r=1,2,\ldots;\ k=2,4,6,\ldots \tag{43}$$

The advantage of using Eq. (43) instead of Eq. (37) is that the effects of noise and interpolation error are minimized. This is because the interpolation errors or spillover effects associated with each modal coordinate tend to cancel each other. Note that for some modes of failure, and depending on the length of the sampling period, the assumption of having inputs constant over two sampling periods may not be correct. It is recommended that both Eqs. (43) and (37) be used to detect failure.

If we consider the case where the same force is applied for two sampling periods, a relation similar to Eq. (43) can be derived for the case of only acceleration measurements. Using a procedure similar to Eqs. (34-37) yields

$$f_r(kT)=f_r(kT-2T)+\frac{\ddot{u}_r(kT-T)}{\cos\omega_r T}-2\ddot{u}_r(kT)+\frac{\ddot{u}_r(kT+T)}{\cos\omega_r T},\quad r=1,2,\ldots,;\ k=2,4,6,\ldots \tag{44}$$

which can also be used to identify the modal excitations.

Let us now consider identification of the external excitations assuming that a number of the modal excitations have been identified by the process described above. As stated earlier, one method is by using Eq. (2b) and replacing the upper limit in the summation by n, where n denotes the number of monitored modes, so that

$$\hat{f}(x,t)=\sum_{r=1}^{n}\phi_r(x)\hat{f}_r(t) \tag{45}$$

where $\hat{f}(x,t)$ denotes the estimate of the force density. The accuracy of estimation depends on the number of monitored modes.

In most cases, however, actuator forces and moments are in the forms of discrete point inputs, whose locations are known. We wish to identify the actuator inputs by making use of these properties and by monitoring as few nodes as possible. To this end, we consider Eq. (26), which relates the modal forces to the actual inputs. A similar relationship can be written for torque inputs. One way of synthesizing the estimates of the actual forces $\boldsymbol{F}(t)$ is by inverting Eq. (26), which gives

$$\hat{\boldsymbol{F}}(kT) = B^{-1}\hat{\boldsymbol{f}}(kT), \quad k=1,2,\ldots \tag{46}$$

where the $(\hat{\ })$ in Eq. (46) denote identified quantities. The simplest way to invert B is to have an adequate number of sensors such that as many modes as the number of external excitations are monitored, so that B becomes a square matrix. Regarding the question as to which modes should be selected for monitoring, we note that the lower modes can be extracted from the sensors output with less error and lower modal noise.[12] In addition, lower natural frequencies further reduce the magnitude of the coefficients used in the identification procedure. It then seems reasonable to monitor the lower modes. Also, monitoring the lowest n modes guarantees B to be nonsingular.[17] It should be noted that having errors in the initial estimate of $f_r(t)$ $(r=1,2,\ldots,n)$ is equivalent to having errors in the initial estimate of $F_j(t)$ $(j=1,2,\ldots,m)$. Likewise, these errors do not propagate as time unfolds.

General Considerations

In the preceding section, a method was outlined to identify the modal forces acting on a distributed system, based on velocity and acceleration measurements or only acceleration measurements. A common form of failure detection is by voting.[6] In detecting actuator failure, voting implies obtaining more than one estimate of the modal forces and comparing these estimates with each other and with the actuator commands. This way, a more reliable detection of failure can be achieved. In the method outlined above, Eqs. (37), (43), and (44) are not the only relations that can be obtained by rearranging the system response. Other relations can easily be obtained to implement a voting procedure. One way is to use the equations of motion in the modal space directly. By examining Eq. (4), we observe that if the modal displacements and modal accelerations associated with the rth mode are known, then $f_r(t)$ can be readily identified.

The disadvantages of using Eq. (4) are that the modal displacements are multiplied by a factor of ω_r^2 and that displacement measurements need to be used (for the case of modal filters), which are less accurate than velocity measurements from hardware considerations. In order to alleviate this and in order to use velocity and acceleration measurements to implement the failure detection, we observe that

$$u_r(t) = u_{r0} + \int_0^t \dot{u}_r(\tau)\,\mathrm{d}\tau, \qquad r=1,2,\ldots \tag{47}$$

where u_{r0} denote initial conditions. Introducing Eqs. (47) into Eq. (4) we obtain

$$f_r(t) = \ddot{u}_r(t) + \omega_r^2\left(\int_0^t \dot{u}_r(\tau)\,\mathrm{d}\tau + u_{r0}\right), \qquad r=1,2,\ldots \tag{48}$$

which can be used to detect failure. Because only velocities at the sampling instances are available, their integration can be carried out as a summation. Assuming that the initial modal displacements are not known, we can estimate the modal forces by

$$\hat{f}_r(kT) = \ddot{u}_r(kT) + \omega_r^2 T \sum_{j=1}^{k} \dot{u}_r(jT),$$
$$k=1,2,\ldots;\ r=1,2,\ldots \tag{49}$$

and we note that the modal velocities are multiplied by a factor of $\omega_r^2 T$, which, depending on the sampling period, can be a very small quantity. It can easily be shown that the propagation of error when Eq. (49) is used is very similar to the propagation of error when Eq. (37) is used to identify the modal excitations. Again, the effects of not knowing initial conditions do not change as time unfolds.

It should be noted that the failure detection procedure, which is based on comparing the actuator commands with the identified external excitations, can be performed as an on- or off-line operation. The computational effort is minimal, and the identification process makes use of the existing hardware and does not require any additional components to detect failure. Also, because plots over time need to be compared, instead of instantaneous comparisons, a need for using artificial intelligence emerges.

Up to now, we considered a failure detection scheme where the actual control inputs are identified by synthesizing the identified modal inputs and then compared with the actuator commands. Even though the modal forces were identified, we did not make use of them to detect failure. One reason for this is because failure of a single actuator affects all the modal coordinates, so that a reliable decision as to which component is faulty cannot be made by inspecting the modal forces alone.

It turns out that use can be made of the identified modal forces to detect failure. Furthermore, a guideline for actuator placement can be developed in a way to aid the failure detection process. Primary considerations in selecting the actuator locations include controllability and effectiveness of the control law. It has been recommended that the actuators be placed as evenly as possible[18] or, for certain methods, at the nodes of uncontrolled modes.[19] Here the following is proposed: If each actuator (say rth) is on one of the nodes of a monitored mode (say jth), then failure of the rth actuator will affect the response of every mode, except the response of the jth mode. Then, if the rth actuator is suspected of being faulty, plots of the identified modal forces can be compared with the designed modal control forces. The designed and identified modal forces should look different, except for the jth mode.

The guideline for actuator location proposed here calls for the placement of each actuator on the nodes of the monitored modes. Note that because the number of monitored modes is selected as the same as the number of actuators, there will always be more nodes than actuators, so that the actuators can be placed evenly throughout the distributed system.

It is easy to show that the procedure described above to identify the external excitations for distributed systems is applicable to discrete systems as well. For a self-adjoint discrete system the equation of motion has the form

$$M\ddot{y}(t) + Ky(t) = \boldsymbol{F}(t) \tag{50}$$

where M and K are symmetric matrices of order $m \times m$ and $y(t)$ and $\boldsymbol{F}(t)$ are output and input vectors, respectively. Solution of the eigenvalue problem yields the eigenvector matrix Q and the modal equations

$$\ddot{\boldsymbol{u}}(t) + \Lambda\boldsymbol{u}(t) = \boldsymbol{f}(t) = Q^T\boldsymbol{F}(t) \tag{51}$$

where $\boldsymbol{u}(t) = Q^T My(t)$ and Λ is a diagonal matrix containing the eigenvalues. Because Eq. (51) has the same form as Eq. (4), we can apply the identification scheme outlined earlier to identify external excitations for discrete systems as well. Note that it is necessary to have as many sensors as the order of the mathematical model, unless an observer is used. On the other hand, problems associated with distributed systems, such as having an infinite number of degrees of freedom and interpolation error, do not exist for discrete systems.

In a failure detection process, after failure is detected and the faulty components are isolated, the next step is to take corrective action. Corrective action can consist of bringing on-line a standby component and recomputing the control

Table 1 The *B* matrix

0.2628	0.3872	0.4472	0.4253	0.3162
0.4253	0.3873	0.0000	−0.2628	−0.4472
0.4253	0.0001	−0.4472	−0.2628	0.3162
0.2628	−0.3872	0.0000	0.4253	0.0000
0.0000	−0.3874	0.4472	0.0000	−0.3162

Table 2 The *CD* matrix for 11 sensors

0.9999	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.9988	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.9940	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.9821	0.0000	0.0000	0.0000	0.2106
0.0000	0.0000	0.0000	0.0000	0.9590	0.0000	0.1594	0.0000

gains or of redesigning the control system using one less actuator. It is clear that the type of control method is very much related to the effort associated with taking corrective action. This issue will be addressed in a subsequent paper.

Numerical Example

As a numerical example, let us consider actuator failure detection during control of a uniform beam pinned at both ends. The beam parameters are chosen as $m(x)=1$, $L=EId^4/dx^4$, and $0<x<\ell$, where EI is the stiffness distribution, taken as $EI=1$, and ℓ the length of the beam chosen as $\ell=10$. The beam admits a closed-form eigensolution in the form

$$\lambda_r=\omega_r^2=(r\pi/\ell)^4,\quad \phi_r(x)=\sqrt{2/\ell}\sin(r\pi x/\ell),\quad r=1,2,\ldots \tag{52}$$

We regulate the motion of the beam by modal control and use modal filters to extract the modal coordinates from the sensors data. By examining the *CD* matrix [Eq. (20)], the accuracy of the modal filters can be assessed.

Five force actuators are placed on the beam at the following locations: 0.2, 0.333, 0.5, 0.6, and 0.75 ℓ. The actuators locations are chosen based on the guideline suggested in the previous section, so that each actuator is on one of the nodes of a monitored mode (fifth, third, second, fifth, and fourth, respectively). In order to identify failure, we observe that five modes need to be monitored and the lowest five modes are selected. Table 1 shows the *B* matrix.

The control law is based on independent modal-space control.[17,18] In the system simulation, it is assumed that the lowest eight modes contribute to the system output, so that we have three uncontrolled modes. The system output is observed by 11 velocity and acceleration sensors that are spaced evenly along the beam. The sensors output is interpolated using quadratic interpolation functions from the finite element method.[12] Table 2 shows the *CD* matrix for 11 sensors and for the monitored modes. Ideally, when distributed measurements are available, the first five columns of *CD* should look like an identity matrix[12] and the last three columns should be zero. As can be seen from Table 2, the last three columns of *CD* have nonzero entries, which implies that the extracted modal coordinates are contaminated by modes that are not monitored. Even though increasing the number of sensors would alleviate this problem,[12] this specific number of sensors was retained in an effort to make the numerical example more realistic.

Failure of the second actuator of 65% is assumed, so that $S_2=0.35$. In addition, the input of the faulty actuator is contaminated by random noise in the form $v_2=N(0,0.1)$. The inputs of the remaining actuators and the outputs of all the sensors are also contaminated by random noise, of the form $N(0,0.025)$. Viscous damping of $\zeta=0.02$ is added to each mode so that the failure detection is carried out in the presence of slight parameter uncertainties as well. A sampling period of 1/40 s is chosen and the same actuator inputs are applied for two sampling periods.

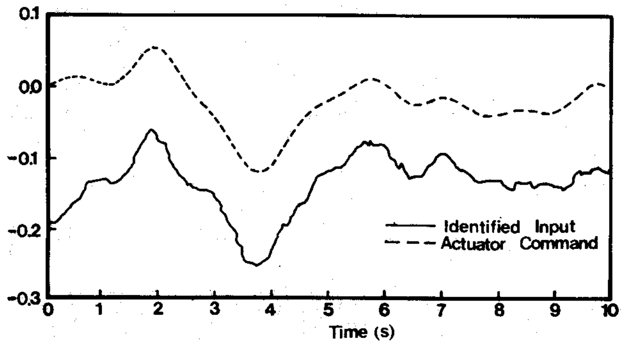

Fig. 1 Identified input vs actuator command for first actuator.

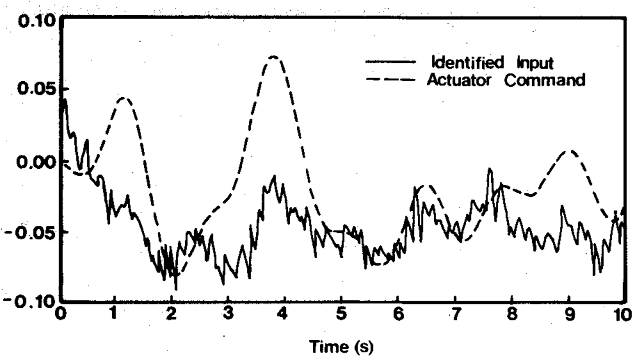

Fig. 2 Identified input vs actuator command for second actuator.

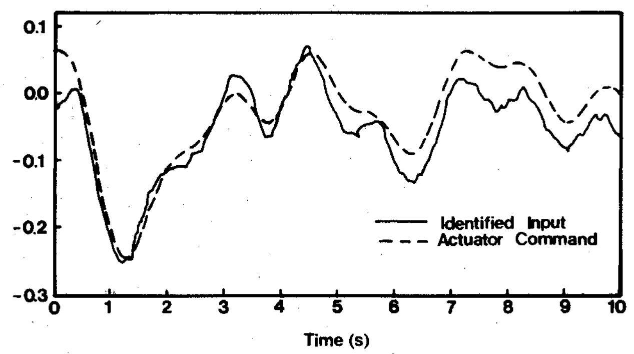

Fig. 3 Identified input vs actuator command for third actuator.

Figures 1-5 compare the identified external inputs with the actuator commands, where Eq. (43) is used to identify the modal forces. As can be observed, there is negligible difference between the identified inputs and actuator commands, except for the second actuator. The small differences are due to measurement and actuator noise, interpolation error, and damping, which are not accounted for in the identification scheme. Also, the variance in the identified second actuator input is larger than the variances in the other identified forces, which is another indication of failure of the second actuator.

We observe from Figs. 1-5 that the difference between the identified excitations and actuator commands gets larger as time evolves. This can be attributed to the presence of damping in the system equations, which can be treated as a source of parameter uncertainties since the identification procedure considers an undamped model. Because of the accumulation of error due to such uncertainties, plots of the identified external disturbances and actuator commands should not be compared for very long time periods.

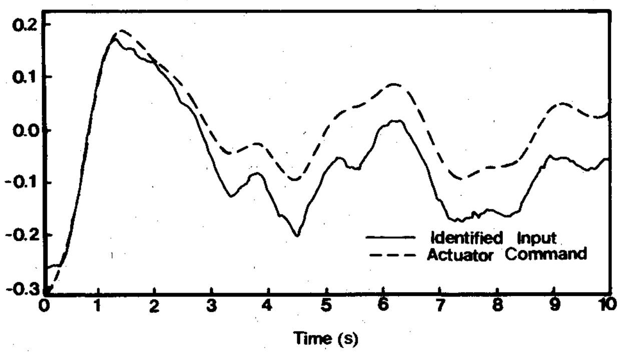

Fig. 4 Identified input vs actuator command for fourth actuator.

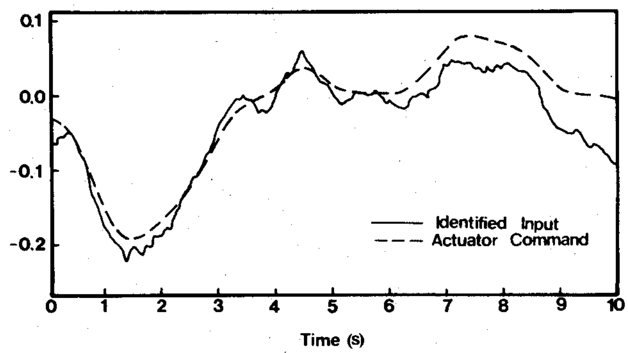

Fig. 5 Identified input vs actuator command for fifth actuator.

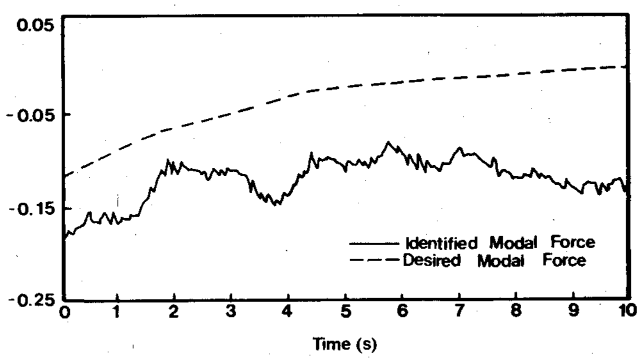

Fig. 6 Identified vs desired first modal force.

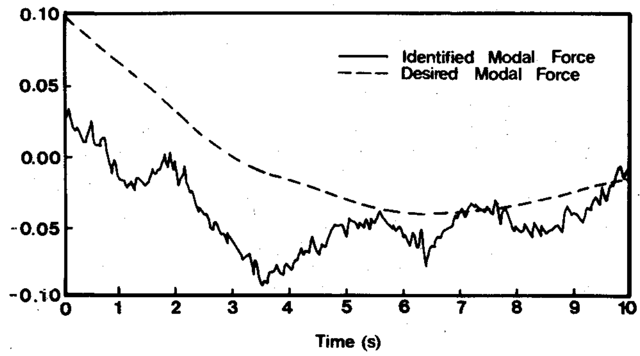

Fig. 7 Identified vs desired second modal force.

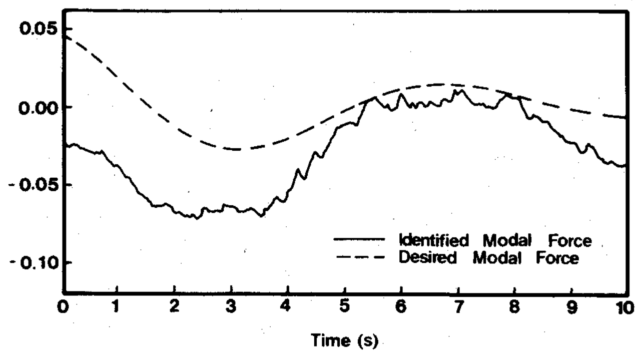

Fig. 8 Identified vs desired third modal force.

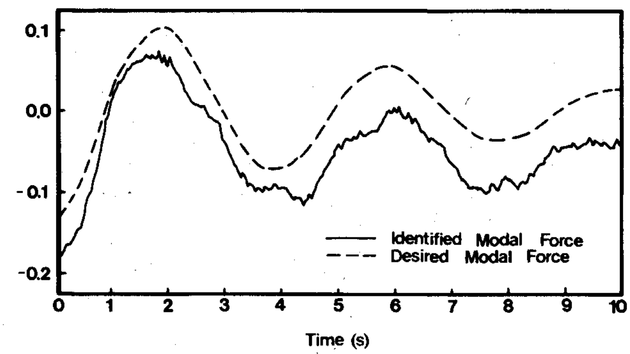

Fig. 9 Identified vs desired fourth modal force.

In order to verify that the second actuator is faulty, we compare the identified modal excitations with the desired modal control inputs. Figures 6-10 plot and compare the identified and desired modal forces. We observe that, for the third, fourth, and fifth modes, there is very little difference between the identified and desired modal forces. Note that the second actuator is located on the first node of the third mode. In the first and second modes, however, there is substantial change, which indicates a difference between the actuator commands and actual external inputs. This change in the first and second modal forces, together with insignificant change in the other modal forces, verifies that an actuator is not functioning properly. The third modal force is not affected by failure. The reason why the fourth and fifth modes are not affected by failure can be explained by noting that it is more difficult to excite the higher modes, so that they are affected less by actuator failure. However, the third mode is the lowest one not affected by failure, which helps in verifying that the second actuator is faulty.

The above results suggest that, when locating the actuators, more of them should be placed on the nodes of the lower monitored modes. It should also be noted that comparing the modal forces alone could not have led to an accurate identification of the faulty components, which implies that the task of failure detection should be in the form of a sequential decision process, where inputs from different evaluations are used to detect and verify failure.

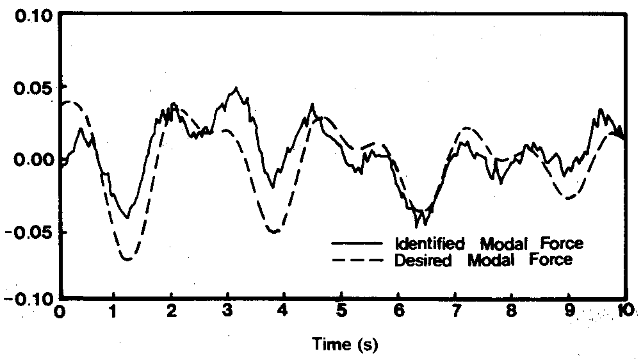

Fig. 10 Identified vs desired fifth modal force.

Conclusions

A method is presented for detecting actuator failures in distributed-parameter systems. The method is based on identifying the modal forces from the responses of the modal velocities and accelerations. After the modal forces are identified, estimates of the external forces are synthesized and compared with the actuator commands to detect failure and isolate the faulty component(s). The failure detection method is applicable to large-order systems, where a substantial number of actuators are used. It can be carried out as an on- or off-line operation and can be implemented with the existing hardware. The effects of actuator failure and factors affecting the accuracy of the failure detection, such as measurement noise and observation spillover, are analyzed. It is shown that colocated control is more desirable for reliability reasons. Also, for modal control, modal filters are found to be more advantageous than observers. A guideline is proposed for locating the actuators in a way to aid the failure detection process.

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